Parameter Estimation for Peaky Altimetric Waveforms

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Abstract – A simple parametric model was recently introduced to model peaky altimetric waveforms. This model assumes that the received altimetric waveform is the sum of a Brown echo and Gaussian peaks. This paper derives the maximum likelihood estimator for the parameters of this Brown + peak model. Simulation results conducted on synthetic and real altimetric waveforms allow one to appreciate the performance of the proposed estimator.

Keywords: altimetry, maximum likelihood, Brown model.

1. NTRODUCTION

The contamination of ocean echoes by land returns or by the summation of backscattered signals coming from separate reflective surfaces considerably damages availability and quality of altimeter waveforms. A great effort is devoted to process these waveforms in order to move the altimetric measurements closer to the coast. In the frame of the CNES/ PISTACH project aiming at improving coastal altimeter products, waveforms are classified according to their geometrical shapes (Thibaut and Poisson, 2008). The goal of this classification is to isolate echoes having similar geometrical characteristics in order to estimate the corresponding altimeter parameters thanks to dedicated retracking algorithms. In this paper, we study an innovative retracking algorithm based on maximum likelihood estimation for waveforms corrupted by the presence of peaks on their trailing edge. The proposed algorithm is based on a mathematical model recently introduced by Gómez-Enri et al. (2009).

2. PROBLEM FORMULATION

A simplified formulation of Brown's model assumes that the received altimeter waveform (after removing the thermal noise that can be estimated from the first data samples) is parameterized by three parameters P_u, τ and σ_c via the following equation

$$x_k = \frac{P_u}{2}\left[1 + erf\left(\frac{kT_s - \tau - \alpha\sigma_c^2}{\sqrt{2}\sigma_c}\right)\right]\exp\left[-\alpha\left(kT_s - \tau - \frac{\alpha\sigma_c^2}{2}\right)\right] \qquad (1)$$

where $x_k = x(kT_s)$ is the kth data sample of the received altimetric signal, T_s is the sampling period, $erf(t) = \left(2/\sqrt{\pi}\right)\int_0^t e^{-z^2}dz$ stands for the Gaussian error function and α is a known parameter depending on the satellite. The parameters P_u and τ appearing in (1) are referred to as amplitude and epoch whereas the third parameter σ_c is related to the significant wave height SWH as follows:

$$\sigma_c^2 = \sigma_p^2 + \left(\frac{SWH}{2c}\right)^2$$

where c denotes the light speed and σ_p^2 is a known parameter depending on the altimeter point target response. The Brown model has been derived for modeling altimeter echoes associated to oceanic surfaces. However, this model is not appropriate to altimetric waveforms backscattered from non-oceanic surfaces such as ice and land or from coastal areas (Gómez-Enri et al. 2009, Thibaut and Poisson, 2008). Indeed, over such surfaces, altimetric echoes can show some peaks in their trailing edge due to backscattering returns from non water areas. A simple mathematical model for altimetric waveforms corrupted by peaks was recently introduced in Gómez-Enri et al. (2009). This model assumes that an altimetric waveform denoted as $\tilde{x}_k$ can be represented as the superposition of a Brown echo x_k and a sum of Gaussian peaks p_k

$$\tilde{x}_k = x_k + p_k$$

where x_k has been defined in (1),

$$p_k = \sum_{i=1}^{q} A_i \exp\left[-\frac{(kT_s - t_i)^2}{2\sigma_i^2}\right] \qquad (2)$$

and q is the number of peaks. Any peak defined in (2) is parameterized by an amplitude A_i, a location parameter t_i and a width related to the variance σ_i^2 with $i=1,\ldots,q$. As a consequence, the unknown parameters for the q peaks are gathered in $\mathbf{A} = (A_1,\ldots,A_q)$, $\mathbf{t} = (t_1,\ldots,t_q)$ and $\boldsymbol{\sigma}^2 = (\sigma_1^2,\ldots,\sigma_q^2)$. A particular attention will be devoted to the case of a single peak (i.e., $q = 1$) in this summary. However, the case of multiple peaks will be addressed in the final paper. Note that the migration of peaks along the trailing edge as in Gómez-Enri et al. (2009) is not considered here. Altimeter data are corrupted by multiplicative speckle noise. In order to reduce the influence of this noise affecting each individual echo, a sequence of L consecutive echoes are averaged on-board. Assuming pulse-to-pulse statistical independence (Walsh, 1982), the resulting speckle noise denoted as n_k is distributed according to a gamma distribution whose shape and scale parameters equal the number of looks L (i.e., the number of incoherent summations of consecutive echoes). When using the Brown + peak model defined previously, an observed altimetric waveform can be expressed as

$$\tilde{y}_k = \tilde{x}_k n_k, \quad k = 1,\ldots,N. \qquad (3)$$

The problem addressed in this paper is the joint estimation of the Brown model parameter vector $\boldsymbol{\theta}_B = (P_u, \tau, \sigma_c)^T$ and the peak parameter vector $\boldsymbol{\theta}_p = (\mathbf{A}, \mathbf{t}, \boldsymbol{\sigma}^2)^T$ using the data samples $\tilde{\mathbf{y}} = (\tilde{y}_1,\ldots,\tilde{y}_N)^T$. We propose to extend the

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maximum likelihood (ML) estimation strategy initially derived by Dumont (1985) for estimating the parameters of the Brown model to the Brown + peak model defined by Gómez-Enri et al. (2009).

3. MAXIMUM LIKELIHOOD ESTIMATION

Since the proposed estimation strategy strongly relies on the ML estimator (MLE) derived in Dumont (1985), this section first recalls briefly some principles of this estimator. The extension to the Brown + peak model is then investigated.

3.1 Maximum Likelihood for Brown's Model

The MLE of $\boldsymbol{\theta}_B$ is classically obtained by differentiating the log-likelihood of the observed data with respect to the unknown parameters P_u, τ and σ_c Kay (1993). Due to the absence of closed-form expressions for the MLE of $\boldsymbol{\theta}_B$, iterative algorithms searching for local maxima of the log-likelihood have been proposed by Dumont (1985). More precisely, the following recursion referred to as the quasi-Newton recursion is currently used for estimating the parameters of the Brown model

$$\boldsymbol{\theta}_B(n+1) = \boldsymbol{\theta}_B(n) - \mu_n \left(\mathbf{B}\mathbf{B}^T\right)^{-1}\mathbf{B}\mathbf{D} \qquad (8)$$

where $\mathbf{B}$ is a 3xN matrix whose elements are $B_{ik} = \frac{1}{x_k}\frac{\partial x_k}{\partial \theta_{B,i}}$ and $\mathbf{D}$ is an N x 1 vector whose elements are $D_k = 1-(y_k/x_k)$ with i = 1,…,3 and k = 1,…,N. This recursion has been implemented in all the current retracking algorithms used in Jason/Envisat ground processing chains.

3.2 Maximum Likelihood Estimation for Brown + Peak Model

We will show in the full paper that the MLE of $\boldsymbol{\theta} = \left(\boldsymbol{\theta}_B^T, \boldsymbol{\theta}_p^T\right)^T$ can be determined by using the following quasi-Newton recursion

$$\boldsymbol{\theta}(n+1) = \boldsymbol{\theta}(n) - \mu_n \begin{pmatrix} \tilde{\mathbf{B}}\tilde{\mathbf{B}}^T & \tilde{\mathbf{B}}\tilde{\mathbf{P}}^T \\ \tilde{\mathbf{P}}\tilde{\mathbf{B}}^T & \tilde{\mathbf{P}}\tilde{\mathbf{P}}^T \end{pmatrix}^{-1} \begin{pmatrix} \tilde{\mathbf{B}} \\ \tilde{\mathbf{P}} \end{pmatrix} \mathbf{D}$$

where $\tilde{\mathbf{B}}$ is a 3xN matrix whose elements are $\tilde{B}_{ik} = \frac{1}{\tilde{x}_k}\frac{\partial x_k}{\partial \theta_{B,i}}$ with i = 1,…,3, k = 1,…,N and $\tilde{\mathbf{P}}$ is a (3q)xN matrix whose elements are $\tilde{P}_{ik} = \frac{1}{\tilde{x}_k}\frac{\partial p_k}{\partial \theta_{p,i}}$ with i = 1,…,3q and k = 1,…,N.

4. SIMULATION RESULTS

Many simulations have been conducted to validate the proposed MLE for the Brown + peak model. The first example considers the sum of a Brown waveform with parameters P_u=60, τ=32 and SWH=2 (σ_p^2 has been adjusted to its nominal value used for Jason-2 satellite) and a Gaussian peak defined by $\boldsymbol{\theta}_p = (A_1, t_1, \sigma_1^2) = (90, 60, 25)$. The resulting synthetic signal has been contaminated by a multiplicative speckle noise with L=100 looks. The proposed quasi-Newton recursion (referred to as PEAK-MLE) has been tested on the resulting echo and has been compared with the usual MLE which ignores the presence of the Gaussian peak. The results depicted in Fig. 1 (top) show that the proposed PEAK-MLE allows one to estimate the parameters of the Brown model as well as the peak parameters, while the classical MLE obviously fails to correctly estimate θ_B. The second example considers a real Jason-2 waveform collected close to Mariana island, between the Philippines and Hawaii. As shown in Fig. 1 (bottom), the proposed estimator allows one to capture the joint characteristics of the Brown waveform and the Gaussian peak for this echo.

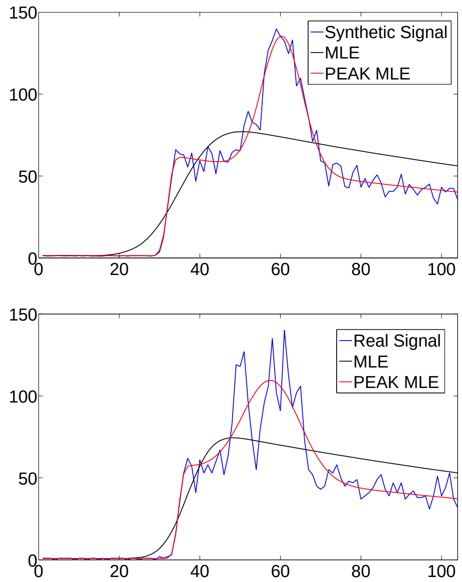

Figure 1. Synthetic Brown + peak signal and its estimations using MLE and PEAK-MLE (top); real JASON-2 waveform and its estimations using MLE and PEAK-MLE (bottom).

5. CONCLUSION

We studied an estimation strategy based on the maximum likelihood principle for a Brown + peak model. The model is appropriate for altimetric signals resulting from the summation of a Brown echo and Gaussian peaks. Next, we shall study the performance of the proposed estimator in terms of mean square error for a database of peaky altimetric signals. This database has been obtained from a classifier studied within the PISTACH project (Thibaut and Poisson, 2008). The generalization of the proposed algorithm to multiple Gaussian peaks and to other shapes of peaks is currently under investigation.

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